Inference rules

M.P.

$$\begin{array}{l} (p \supset q) \\ p \\ \hline /\therefore q \end{array}$$

M.T.

$$\begin{array}{l} (p \supset q) \\ \sim q \\ \hline /\therefore \sim p \end{array}$$

H.S.

$$\begin{array}{l} (p \supset q) \\ (q \supset r) \\ \hline /\therefore (p \supset r) \end{array}$$

Simp.

$$\begin{array}{l} (p \bullet q) \\ \hline /\therefore p \end{array} \qquad \begin{array}{l} (p \bullet q) \\ \hline /\therefore q \end{array}$$

Conj.

$$\begin{array}{l} p \\ q \\ \hline /\therefore (p \bullet q) \end{array}$$

D.S.

$$\begin{array}{l} (p \vee q) \\ \sim p \\ \hline /\therefore q \end{array} \qquad \begin{array}{l} (p \vee q) \\ \sim q \\ \hline /\therefore p \end{array}$$

Add.

$$\begin{array}{l} p \\ \hline /\therefore (p \vee q) \end{array} \qquad \begin{array}{l} q \\ \hline /\therefore (p \vee q) \end{array}$$

Dil.

$$\begin{array}{l} (p \supset q) \\ (r \supset s) \\ (p \vee r) \\ \hline /\therefore (q \vee s) \end{array}$$

D.N.

$p :: \sim\sim p$

Comm.

$(p \vee q) :: (q \vee p)$

$(p \bullet q) :: (q \bullet p)$

Assoc.

$((p \vee q) \vee r) :: (p \vee (q \vee r))$

$((p \bullet q) \bullet r) :: (p \bullet (q \bullet r))$

Dup.

$p :: (p \vee p)$

DeM.

$\sim(p \vee q) :: (\sim p \bullet \sim q)$

$\sim(p \bullet q) :: (\sim p \vee \sim q)$

B.E.

$(p \equiv q) :: ((p \supset q) \bullet (q \supset p))$

Contrap.

$(p \supset q) :: (\sim q \supset \sim p)$

C.E.

$(p \supset q) :: (\sim p \vee q)$

Exp.

$((p \bullet q) \supset r) :: (p \supset (q \supset r))$

Dist.

$(p \bullet (q \vee r)) :: ((p \bullet q) \vee (p \bullet r))$

$(p \vee (q \bullet r)) :: ((p \vee q) \bullet (p \vee r))$

C.P.

$$\begin{array}{l} \rightarrow p \\ \quad . \\ \quad . \\ \quad . \\ \quad q \\ \hline /\therefore (p \supset q) \end{array}$$

I.P.

$$\begin{array}{l} \rightarrow p \\ \quad . \\ \quad . \\ \quad . \\ \quad (q \bullet \sim q) \\ \hline /\therefore \sim p \end{array}$$

Q.N.

$\sim(x)\phi x :: (\exists x)\sim\phi x$

$\sim(\exists x)\phi x :: (x)\sim\phi x$

$\sim(x)\sim\phi x :: (\exists x)\phi x$

$\sim(\exists x)\sim\phi x :: (x)\phi x$

C.Q.N.

$\sim(x)(\phi x \supset \psi x) :: (\exists x)(\phi x \bullet \sim\psi x)$

$\sim(\exists x)(\phi x \bullet \psi x) :: (x)(\phi x \supset \sim\psi x)$

$\sim(x)(\phi x \supset \sim\psi x) :: (\exists x)(\phi x \bullet \psi x)$

$\sim(\exists x)(\phi x \bullet \sim\psi x) :: (x)(\phi x \supset \psi x)$

U.I.

$$\begin{array}{l} (x)\phi x \\ \hline /\therefore \phi a \end{array}$$

E.I.

$$\begin{array}{l} (\exists x)\phi x \\ \hline /\therefore \phi a \end{array}$$

provided we flag a

U.G.

$$\begin{array}{l} \rightarrow \text{flag } a \\ \quad . \\ \quad . \\ \quad \phi a \\ \hline /\therefore (x)\phi x \end{array}$$

E.G.

$$\begin{array}{l} \phi a \\ \hline /\therefore (\exists x)\phi x \end{array}$$