

Handout for Section 1.1

Problem Solving Strategies and Principles

In this section we will explore different problem solving strategies and principles that will help you in this class and in your life. Life is about decisions, problems and solutions. While this class is a math class and hence will involve the explicit use of the strategies in math problems. The strategies that we discuss here can be used when writing research papers, performing science experiments, deciding on a major purchase, or investing for your future.

First we'll discuss the process of problem solving. Problem solving involves: preparation, thinking time, insight, and verification.

- Preparation: _____
- Thinking Time: _____
- Insight: _____
- Verification: _____

Now, we'll outline some of the problem solving strategies from the book and then try to use them to solve problems.

STRATEGIES

- **Draw Pictures:** Problems usually contain several conditions that must be satisfied. You will find it useful to draw pictures to understand these conditions before trying to solve the problem.
- **Choose Good Names for Unknowns:** It is a good practice to name the objects in a problem so you can remember their meaning easily.
- **Be Systematic:** If you approach a situation in an organized, systematic way, frequently you will gain insight into the problem.
- **Look for Patterns:** If you can recognize a pattern in a situation you are studying, you can often use it to answer questions about that situation.
- **Try a Simpler Version of the Problem:** You can begin to understand a complex problem by solving some scaled-down versions of the problem. Once you recognize a pattern in the way you are solving the simpler problems, then you can carry over this insight to attack the full-blown problem.
- **Guessing is Okay:** One of the difficulties in solving word problems is that you can be afraid to say something that may be wrong and consequently sit staring at the problem, writing nothing until you have the full-blown solution. Making guesses, even incorrect guesses, is not a bad way to begin. It may give you some understanding of the problem. Once you make a guess, evaluate it to see how close you are to meeting all the conditions of the problem.
- **Convert a New Problem to an Older One:** An effective technique in solving a new problem is to try to connect it with a problem you have solved earlier. It is often possible to rewrite a condition so the problem becomes exactly like one you have seen before.

There are also some principles that make problem solving easier.

PRINCIPLES:

- **The Always Principle:** When we say a statement is true in mathematics we are saying that statement is true 100 percent of the time. One of the great strengths of mathematics is that we do not deal with statements that are "sometimes true" or "usually true".
- **The Counterexample Principle:** An example that show that a mathematical statement fails to the true is called a *counterexample*. Be careful to understand that when we say a statement is false, we are not saying that it is always false. We are saying that the statement is *not always true*. That is, we can find at least one instance in which it is false.
- **The Order Principle:** When you read mathematical notation, pay careful attention to the *order* in which the operations must be performed. The order in whihc we do things in mathematics is as important as in everyday life. Putting you socks on before your shoes is quite different than putting your shoes on before your socks.
- **The Splitting Hairs Principle:** You should "split hairs" when reading mathematical terminology. If two terms are similar but sound slightly different, they usually don't mean the same thing. For example, the words "equal" and "equivalent" are interchangeable in everyday english, but very different in mathematics. You should also watch for slight differences in mathematical notation.
- **The Analogies Principle:** Much of the formal terminology taht we use in mathematiacs sounds like words that we use in everyday life. This is not a coincidence. Whenever you can associate ideas from real life with mathematical concepts, you will better understand the meaning behind the mathematics you are learning.
- **The Three-Way Principle:** Whether you are learning a new concept or trying to gain insight intoa problem, it is helpful to use the ideas we have discussed in this chapter to approach mathe-matical situations in three ways.
 - *Verbally* - Make analogies. State the problem in your own words. Compare it with situations you have seen in other areas of mathematics.
 - *Graphically* - Drawa graph. Draw a diagram.
 - *By example* - Make numerical or other kinds of examples to illustrate the situation.